Symbolizing Conditionals • "if...then" = arrow →

Standard Logical Synonyms for "if...then"

when	provided that	allowing that	results in	} These last three can be tricky. When you see them, it's best to restate the sentence in standard "if...then" language.
whenever	supposing that	in the event that	brings about	
should	assuming that	so long as	leads to	
in case of	given that	as long as		

I. Symbolize the following sentences. (Answers in class or on video.)

1. If you BELIEVE it, then you can ACHIEVE it.
2. In case of FIRE, use STAIRS.
3. Catch ME if YOU can.
4. Everything happens for a REASON, assuming that the world is LOGICAL.
5. Should you FIND a lost wallet, you should RETURN it. (think about the two "shoulds")
6. If I had KNOWN it then, then I would have COMPLAINED. (think about the two "thens")
7. Eating too much JELLO results in feelings of ARTIFICIALITY.
8. Keeping one's cat in the REFRIGERATOR leads to PROBLEMS.
9. The DESIRE for different icings on each individual piece of cake brings about bitter STRESS amongst the partners of highly successful organizations.

The following sentences require the ampersand and the use of parentheses.

10. Whenever you're ANGRY you should SCREAM and SHOUT.
    (Note: Occasionally, when two capitalized words begin with the same letter, I'll underline the second letter of the second word, in which case you should use the underlined letter for symbolization.)
11. You may DISTRIBUTE copies of the source code, provided that you include a copyright NOTICE and a disclaimer of WARRANTY.
12. Supposing that the forces are REPULSIVE and that they follow the LAW of inverse squares, the particle DESCRIBES a hyperbolic orbit round the nucleus and its deflexion can be simply CALCULATED. (From Ernest Rutherford in a 1914 paper "On the Structure of the Atom.")
13. HAPPINESS leads to SUCCESS, and a new pair of BOOTS leads to happiness.
    (Note: we will adopt the convention that when the same simple sentence appears more than once in a longer sentence, it will only be capitalized in its first appearance, even though it is likely to be needed twice in the symbolization. This sentence requires 2 H's.)
14. Janet will ATTEND as long as they serve JELLO, and if they serve MALT liquor we'll see BRENDA and IRENE.

Each of the remaining sentences is slightly unusual for one reason or another.

15. I would have SENT you a card, had I REMEMBERED your birthday.
16. By ENDING waste we can RESOLVE the crisis.
17. One thing that leads students to STRUGGLE is thinking too MUCH.
18. If you work HARD, and if the tests are FAIR, then you will do WELL.

II. Main Connective Exercise: Circle the main connective in the following formulas.

1. $A \rightarrow B$
2. $(R \rightarrow S) \rightarrow T$
3. $G \rightarrow (H \rightarrow I)$
4. $(M \rightarrow N) \rightarrow (R \rightarrow S)$
5. $(J \rightarrow (K \rightarrow L)) \rightarrow M$
6. $(A \& B) \rightarrow (C \& D)$
7. $A \& ((B \rightarrow C) \& D)$
8. $(A \& (B \rightarrow C)) \& D$
9. $R \& ((S \rightarrow T) \rightarrow W)$
10. $(R \rightarrow (S \rightarrow T)) \& W$
11. $A \rightarrow ((B \rightarrow C) \rightarrow (D \rightarrow E))$
12. $(B \& R) \& ((A \rightarrow B) \& E)$
13. $((M \rightarrow R) \rightarrow (I \rightarrow P)) \& S$
14. $(E \& ((F \rightarrow N) \& D)) \rightarrow (C \& M)$
15. $(T \& W) \rightarrow [H \& (M \rightarrow (A \rightarrow Z))]$

For additional info and practice see ifpthenq.net Online materials are numbered in correspondence to the pages of this text.

Symbolizing Conjunctions • "and" = ampersand &

Standard Logical Synonyms for "and"

but	yet	although	} Note: These last four tend to appear at the beginning of a sentence, and work with a comma to place an ampersand.
however	still	even though	
moreover	as well as	though	
nevertheless	; (semicolon)	while	
nonetheless	, (comma)		

For symbolization with ampersands parentheses are optional. These are all equivalent:

A&B&C&D
(A&B)&(C&D)
((A&B)&C)&D
A&((B&C)&D)

I. Symbolize the following sentences. (Answers in class or on video.)

1. The beer was WARM, FLAT and BITTER, but at least it contained ALCOHOL.
2. Although I ENJOYED the story, it was both DISTURBING and PERPLEXING.
3. Given that your KIDS like macaroni and cheese, they should also try FETTUCINE alfredo and LASAGNA.
4. While it's true that if you eat the JELLO you'll be HAPPY, you'll be DISMAYED if you try the PUDDING.
5. A COLLEGE education leads to a better JOB and a higher SALARY, however, one should care about more than MONEY.
6. If the LOGICIANS win the competition, the POETS will be angry, but the MATHEMATICIANS will be heartbroken.
7. It's my PARTY, and I'll CRY if I WANT to. (Leslie Gore 1963)
8. PLATO and ARISTOTLE will stay home if it's RAINING, nevertheless, the conference will be ENTERTAINING, provided that Zeno talks about TIME and MOTION.
9. When it RAINS it POURS and when he SLEEPS he SNORES, but if you LISTEN very carefully you can hear the subtle WHIRRING and CLICKING of the machinery of existence.
◊10. We're pleased that you're COMING; let us know the DATE and we'll SCHEDULE a tour.

II. Use the key to create sentences that could be symbolized by the formulas. (Answers below.)

J = Janet makes jello I = Irene dances the tango E = Elisa plays the kazoo
P = Janet makes pretzels B = Brenda dances the tango

1. J&P
2. (I&B)&E use "although"
3. (J&E)→B
4. (J→I)&(P→B)
5. (J&P)→B
6. J&(P→B)
7. J→(P&B)
8. (J→P)&B
9. I→(B→(J&P))
10. (E→J)&(J→B)
11. E→(J&P) use "results in"
12. (I&B)→(E→J)

Answers: These answers are not unique; there are many others which would also be correct.

1. Janet makes jello and pretzels.
2. Although Irene and Brenda dance the tango, Elisa plays the kazoo.
3. If Janet makes jello and Elisa plays the kazoo, then Brenda will dance the tango.
4. If Janet makes jello then Irene will dance the tango, but if Janet makes pretzels then Brenda will dance the tango.
5. If Janet makes jello and pretzels, then Brenda will dance the tango.
6. Janet WILL make jello, but if she makes pretzels then Brenda will dance the tango.
7. If Janet makes jello, then she'll make pretzels and Brenda will dance the tango.
8. If Janet makes jello then she'll make pretzels, but (regardless of what Janet does) Brenda will dance the tango.
9. If Irene dances the tango, then if Brenda dances the tango (too), Janet will make jello and pretzels.
10. If Elisa plays the kazoo, then Janet will make jello, and if Janet makes jello, then Brenda will dance the tango.
11. Elisa's playing the kazoo will result in Janet's making jello and pretzels.
12. If Irene and Brenda dance the tango, then if Elisa plays the kazoo, Janet will make jello.

Symbolization • More Conditionals and Conjunctions

I. Symbolize the following sentences. (Answers below.)

1. The cat is the world's most EVIL creature; it kills for PLEASURE and AMUSEMENT.
2. In the event that you are eating HEDGEHOGS, you must serve them with TEA and WATERCRESS.
3. Though the TRUTH will set you free, GASOLINE will set you on fire.
4. He hated BUNNIES, moreover, he was indifferent to JELLO and PUDDING.
5. As long as JANET and BRENDA are laughing, you are probably SAFE.
6. Although I enjoy TOAST, OATMEAL is more nourishing and PANCAKES are more round.
7. The audience will LAUGH and CHEER, and the STUDIO will plan a sequel, if the movie is GOOD.
8. When the WIND blows the cradle will ROCK, and when the BOUGH breaks the cradle will FALL.
9. While logic may BREAK your heart, love will DESTROY your mind and TORTURE your soul.
10. Being TIRED and HUNGRY during a test results in LOGICAL errors and lousy GRADES.
11. Given that the WORLD is more beautiful than it seems then I may be HAPPIER than I feel, and if I could run as fast as a CHEETAH then I could eat ZEBRAS and ANTELOPE.
12. SCRATCHY, ITCHY, REN and STIMPY are all cartoon characters, NIETZSCHE and HEIDEGGER, however, are philosophers.
13. This is the first day of the rest of my LIFE, and I will use it for STUDYING logic if I WANT to.
14. If BILL drinks all the beer, then DALE will be angry, but HANK and BOOMHAUER will be irate.
15. Watching MOVIES and surfing the INTERNET are both pleasant activities, yet, when they have a CHOICE most students would prefer to SYMBOLIZE sentences.
16. It's a beautiful SUMMER day, nevertheless, if you LISTEN closely you will hear a BUTTERFLY furiously flapping its wings attempting to generate a hurricane.
17. Supposing that he CAN he WILL, and provided that he DOES it'll be GREAT, but you will LAUGH when you HEAR the circumstances.
18. She will be SATISFIED as long as they serve TEA and BISCUITS as well as JELLO.
19. Even though he had always been fond of small furry CREATURES, when the KITTENS began falling from the sky and the HAMSTERS exploded, he realized that nature could be PERILOUS.
20. If FEAR leads to HATE, and hate brings about EVIL, then if I were GOD I'd stop SCARING people. (Remember that when simple components are repeated they are only capitalized in their first appearance.)
21. Supposing that the pastrami is DELICIOUS but too EXPENSIVE, while the TUNA salad is leftover from last week, then you should have the PEANUT butter and jelly.
22. When the moon is NEW and the stars are BRIGHT, he stares at the SKY and contemplates the UNIVERSE, but he transforms into a WOLF when the moon is FULL.
23. By REFLECTING deeply one GAINS awareness of self; by gaining awareness of self one becomes DISTURBED by one's flaws and FEARFUL of mortality. (Philosophy is dangerous.)
24. One thing that leads her to DIFFICULTIES is hording CATS, nonetheless, we all admire IRENE.
25. If the FRUIT is ripe, and if the WINE is plentiful, then we shall celebrate our SUCCESS.

Answers. In a string of ampersands parentheses are optional. In the answers I don't include unnecessary parentheses, unless they seem particularly helpful. Also, remember that the "&" is commutative so p&q = q&p.)

1. E&P&A
2. H→(T&W)
3. T&G
4. B&J&P
5. (J&B)→S
6. T&O&P
7. G→(L&C&S)
8. (W→R)&(B→F)
9. B&D&T
10. (T&H)→(L&G)
11. (W→H)&(C→(Z&A))
12. S&I&R&T&N&H
13. L&(W→S)
14. B→(D&H&O)
15. (M&I)&(C→S)
16. S&(L→B)
17. (C→W)&(D→G)&(H→L)
18. (T&B&J)→S
19. C&((K&H)→P)
20. ((F→H)&(H→E))→(G→S)
21. ((D&E)&T)→P
22. ((N&B)→(S&U))&(F→W)
23. (R→G)&(G→(D&F))
24. (C→D)&I
25. (F&W)→S

Validity and Soundness

I. Say whether the following arguments are **sound**, **valid but not sound**, or **invalid**. (Answers Below)

1. If Kaa is a snake,
then Kaa is a reptile. (T)
Kaa is a snake. (T)
Kaa is a reptile.

2. If Kaa is a reptile,
then Kaa is a snake. (F)
Kaa is a reptile. (T)
Kaa is a snake.

3. If Kaa is a snake,
then Kaa is a reptile. (T)
Kaa is a reptile. (T)
Kaa is a snake.

4. If Kaa is a reptile,
then Kaa is a snake. (F)
Kaa is a snake. (T)
Kaa is a reptile.

5. If Kaa is a snake,
then Kaa can fly. (F)
Kaa can fly. (F)
Kaa is a snake.

6. If Kaa is a cow,
then Kaa can fly. (F)
Kaa is a cow. (F)
Kaa can fly.

7. If the bough breaks,
then the cradle falls. (T)
The bough breaks. (T)
The cradle falls.

8. If the bough breaks,
then the cradle falls. (T)
The cradle fell. (T)
The bough broke.

9. If you are happy,
then you are tall. (F)
You are tall. (T)
You are happy.

10. If you are happy,
then you are tall. (F)
You are happy. (T)
You are tall.

11. If you are a student
then you must study. (T)
You are a student. (T)
You must study.

12. If you are a communist,
then you get free jello. (F)
You are a communist. (F)
You get free jello.

II. Create examples: 1) Taking 1-3 above as a model, use "kangaroo," "marsupial" and "Matilda" to construct a set of three examples, one that's sound, one that's valid but not sound, and one that's invalid. 2) Create your own set of three examples.

III. Say whether the following are True or False.(Answers Below)

1. ___ All valid arguments are sound.
2. ___ All sound arguments are valid.
3. ___ Some invalid arguments are sound.
4. ___ Some premises are valid.
5. ___ All invalid arguments have false conclusions.
6. ___ All valid arguments have true conclusions.
7. ___ All sound arguments have true conclusions.
8. ___ All valid arguments have at least one true premise.

Definitions (Memorize These)

Validity: An argument is valid if and only if:
If the premises are true, then the conclusion must be true.

Soundness: An argument is sound if and only if:
It's valid and all its premises are (actually) true.

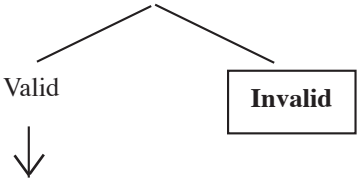
Note that the definition of validity is peculiar because it's a conditional. The definition does NOT say that the premises have to be true, it expresses a truth preserving relation between the premises and the conclusion, such that IF the premises were true, then the conclusion would also have to be true. Validity refers to good structure, and an argument can have good structure even if all its premises and its conclusion are actually false. Look at 12 above; every sentence is false, but IF the premises were true, the conclusion would be as well.

Analyzing an Argument for Soundness and Validity

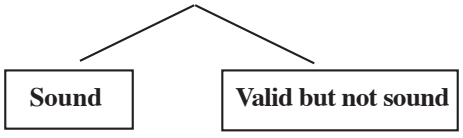
① Structure: Is it **valid** or **invalid**? To determine this you can use either of two methods:

a) Formal method: Does the argument follow a known good pattern? If yes, then it's valid, if no, then it's invalid

b) Intuitive method: Does the argument satisfy the definition of validity? That is, if the premises were taken to be true, would the conclusion also have to be true? If yes, then it's valid, if no, then it's invalid.



② Truth: Is it **sound** or **valid but not sound**? To determine this you need to know the actual truth values of the premises (on this page those values are given in parentheses after the sentences). If all premises are true, then the arg. is sound. If even one premise is false, then it's valid but not sound.



Answers

Section I

1. sound
2. valid b.n.s.
3. invalid
4. invalid
5. invalid
6. valid b.n.s.
7. sound
8. invalid
9. invalid
10. valid b.n.s.
11. sound
12. valid b.n.s.

Section III

1. False
2. True
3. False
4. False
5. False
6. False
7. True
8. False

More Valid Patterns

All the **valid** arguments on the previous page have the same logical structure. That is, they follow the same pattern: "If p then q. p (repeated). Therefore q." But this is only one of many valid patterns. For the arguments below, you'll have to use your intuitions and apply the definition of validity. For each example ask yourself, "If all the premises were true, would the conclusion also have to be true?" If the answer is yes, then the argument is valid. Don't worry if you're not very good at this now; the arguments below preview material from the entire semester.

Say whether the following arguments are **valid** or **invalid**. (Don't worry about soundness.)

If it's cold and wet, then we'll stay home.
It's cold.
We'll stay home.

1. _____

If it's cold or wet, then we'll stay home.
It's cold.
We'll stay home.

2. _____

If it's cold and wet, then we'll stay home.
We stay home.
It's cold and wet.

3. _____

We stay home if and only if it rains.
It's raining.
We're staying home.

4. _____

If it rains, then we'll stay home.
If we stay home, then we'll make jello.
If it rains, then we'll make jello.

5. _____

If it rains, then we'll stay home.
If we stay home, then we'll make jello.
If we're making jello, then it's raining.

6. _____

If you're happy, then you smile.
You're happy.
You smile.

7. _____

If you're happy, then you smile.
You're not happy.
You don't smile.

8. _____

If you're happy, then you smile.
You don't smile.
You're not happy.

9. _____

Hank lives in Arlen.
Dale lives in Arlen.
Hank and Dale live in Arlen.

10. _____

Cartman lives in South Park.
Cartman lives in South Park and Bart lives in Springfield.

11. _____

Stan lives in South Park or Arlen.
Stan doesn't live in Arlen.
Stan lives in South Park.

12. _____

They killed Kenny.
They killed Kenny.

13. _____

Stan and Cartman live in South Park.
Cartman lives in South Park.

14. _____

Stan lives in South Park or Arlen.
Stan lives in South Park.

15. _____

Homer lives in Springfield.
Homer lives in Springfield or Arlen.

16. _____

All men are mortal.
Socrates is a man.
Socrates is mortal.

17. _____

Some men are philosophers.
Socrates is a man.
Socrates is a philosopher.

18. _____

No snakes can fly.
Kaa is a snake.
Kaa cannot fly.

19. _____

Only snakes can fly.
Kaa can fly.
Kaa is a snake.

20. _____

If you study, then you'll pass.
If you pass, then you'll graduate.
If you graduate, then you'll get a good job.
If you get a good job, then you'll make lots of money.
If you make lots of money, then you'll be happy.
If you study, then you'll be happy.

21. _____

All logicians are philosophers.
All philosophers like jello.
Augustus De Morgan is a logician.
Augustus De Morgan likes jello.

22. _____

All cobras are venomous.
All venomous things are dangerous.
Children shouldn't play with cobras.

23. _____

Answers

1. Invalid
2. Valid
3. Invalid
4. Valid
5. Valid
6. Invalid
7. Valid
8. Invalid
9. Valid
10. Valid
11. Invalid
12. Valid
13. Valid
14. Valid
15. Invalid
16. Valid
17. Valid
18. Invalid
19. Valid
20. Valid
21. Valid
22. Valid
23. Invalid as stated, but you can fix it by adding an obvious premise.

First Proof Rule • →out

The Rule →out

$$\begin{array}{l} p\rightarrow q \\ p \\ \hline q \end{array}$$

A good way to read/understand the rule:

If I can find p (on a line by itself) then I can write q.

$$\begin{array}{l} \text{I find p.} \\ \hline \text{I write q.} \end{array}$$

Notes

1. What the rule means: If you have a conditional on one line, and its antecedent on another, you can write its consequent on a third.
2. For all of our rules, if you have what's above the "therefore bar," you may write what's below it.
3. Lower case p and q are merely place holders for actual formulas. Thus p→q stands for any formula that has an arrow as it's main connective. Example: For (A→B)→(C→D): A→B is p, and C→D is q.
4. The traditional Latin name for →out is Modus Ponens.

I. Construct proofs for the following arguments.

Easy

•1. N→M, N, M→R ⊢ R
•2. I→S, S→K, I ⊢ K
•3. A, B→D, A→B, D→E ⊢ E
•4. H→G, G→F, W, W→H ⊢ F
•5. A→(Z→N), A, Z ⊢ N
•6. T, S→(T→W), S ⊢ W

Medium

•7. (T→S)→(H→I), T→S, (H→I)→(R→N) ⊢ R→N
•8. A→B, Y, (A→B)→(Y→Z) ⊢ Z
*9. (F→G)→(G→H), F→G, F ⊢ H
•10. (C→D)→E, E→C, C→D ⊢ D
•11. R→(A→B), R→(B→H), R, A ⊢ H
•12. D, D→(E→F), D→E, F→(M→Z) ⊢ M→Z

Somewhat More Medium

•13. (M→N)→(B→(D→E)), B, M→N, D ⊢ E
•14. R→[S→(T→(W→Z))], R, T, W, S ⊢ Z
•15. E→(E→(E→M)), E, (E→M)→(M→E) ⊢ M→E
•16. B, (B→(A→B))→((A→B)→E), B→(A→B), E→((A→B)→F) ⊢ F
*17. (B→E)→(F→(F→E)), (E→F)→(B→E), B→(E→F), (B→(E→F))→B ⊢ F→E
•18. (J→B)→((F→H)→A), (J→B)→(F→H), (F→H)→J, J→(A→(B→F)), [(J→B)→((F→H)→A)]→(J→B) ⊢ H
*19. S→((H→M)→R), (M→(H→M))→((H→M)→S), M, M→(H→M) ⊢ R

• Solution next page.
* Video Online

II. Imagine that the following pairs of formulas are the first two lines of a proof. What could be derived from them using arrow out? Answers below. For additional problems see the online exercises.

a. 1. A→B
2. A

b. 1. A→B
2. B

c. 1. T
2. M→T

d. 1. T
2. T→M

e. 1. A→(B→E)
2. A

f. 1. (A→B)→E
2. A

g. 1. A→(B→E)
2. A→B

h. 1. (A→B)→E
2. A→B

i. 1. M→T
2. (T→M)→Z

j. 1. T→M
2. (T→M)→Z

k. 1. T
2. T→(M→Z)

l. 1. M
2.T→(M→Z)

m. 1. (A→B)→(M→N)
2. M→N

n. 1. (A→B)→(M→N)
2. A→B

o. 1. A→B
2. A→(B→(M→N))

p. 1. A
2. A→(B→(M→N))

q. 1. (A→B)→M
2. ((A→B)→M)→N

r. 1. A→(B→M)
2. A→((B→M)→N)

s. 1. A→(B→M)
2. (A→(B→M))→N

t. 1. ((A→B)→M)→N
2. A→(B→M)

Answers

a. B	e. B→E	i. Nothing	m. Nothing	q. N
b. Nothing	f. Nothing	j. Z	n. M→N	r. Nothing
c. Nothing	g. Nothing	k. M→Z	o. Nothing	s. N
d. M	h. E	l. Nothing	p. B→(M→N)	t. Nothing

Proofs • → out • Answers

1.

1. N→M	A	
2. N	A	
3. M→R	A	
4. M	1,2	→out
5. R	3,4	→out

2.

1. I→S	A	
2. S→K	A	
3. I	A	
4. S	1,3	→out
5. K	2,4	→out

3.

1. A	A	
2. B→D	A	
3. A→B	A	
4. D→E	A	
5. B	1,3	→out
6. D	2,5	→out
7. E	4,6	→out

4.

1. H→G	A	
2. G→F	A	
3. W	A	
4. W→H	A	
5. H	3,4	→out
6. G	1,5	→out
7. F	2,6	→out

5.

1. A→(Z→N)	A	
2. A	A	
3. Z	A	
4. Z→N	1,2	→out
5. N	3,4	→out

6.

1. T	A	
2. S→(T→W)	A	
3. S	A	
4. T→W	2,3	→out
5. W	1,4	→out

7.

1. (T→S)→(H→I)	A	
2. T→S	A	
3. (H→I)→(R→N)	A	
4. H→I	1,2	→out
5. R→N	3,4	→out

8.

1. A→B	A	
2. Y	A	
3. (A→B)→(Y→Z)	A	
4. Y→Z	1,3	→out
5. Z	2,4	→out

10.

1. (C→D)→E	A	
2. E→C	A	
3. C→D	A	
4. E	1,3	→out
5. C	2,4	→out
6. D	3,5	→out

11.

1. R→(A→B)	A	
2. R→(B→H)	A	
3. R	A	
4. A	A	
5. A→B	1,3	→out
6. B→H	2,3	→out
7. B	4,5	→out
8. H	6,7	→out

12.

1. D	A	
2. D→(E→F)	A	
3. D→E	A	
4. F→(M→Z)	A	
5. E→F	1,2	→out
6. E	1,3	→out
7. F	5,6	→out
8. M→Z	4,7	→out

13.

1. (M→N)→(B→(D→E))	A	
2. B	A	
3. (M→N)	A	
4. D	A	
5. B→(D→E)	1,3	→out
6. D→E	2,5	→out
7. E	4,6	→out

14.

1. R→[S→(T→(W→Z))]	A	
2. R	A	
3. T	A	
4. W	A	
5. S	A	
6. S→(T→(W→Z))	1,2	→out
7. T→(W→Z)	5,6	→out
8. W→Z	3,7	→out
9. Z	4,8	→out

15.

1. E→(E→(E→M))	A	
2. E	A	
3. (E→M)→(M→E)	A	
4. E→(E→M)	1,2	→out
5. E→M	2,4	→out
6. M→E	3,5	→out

16.

1. B	A	
2. (B→(A→B))→((A→B)→E)	A	
3. B→(A→B)	A	
4. E→((A→B)→F)	A	
5. A→B	1,3	→out
6. (A→B)→E	2,3	→out
7. E	5,6	→out
8. (A→B)→F	4,7	→out
9. F	5,8	→out

18.

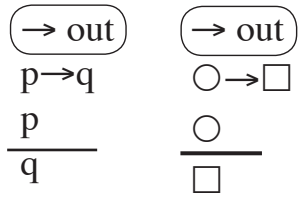
1. (J→B)→((F→H)→A)	A	
2. (J→B)→(F→H)	A	
3. (F→H)→J	A	
4. J→(A→(B→F))	A	
5. [(J→B)→((F→H)→A)]→(J→B)	A	
6. J→B	1,5	→out
7. (F→H)→A	1,6	→out
8. F→H	2,6	→out
9. J	3,8	→out
10. A	7,8	→out
11. A→(B→F)	4,9	→out
12. B	6,9	→out
13. B→F	10,11	→out
14. F	12,13	→out
15. H	8,14	→out

Notes:

1. For problems 1-10 on this page your answer should be identical to what's shown. But for the others it's possible for some lines to appear in a different order. For instance, in proof 11 above it's possible for the order of lines 5, 6 and 7 to vary.

2. I always write justification numbers in numerical order, but this isn't necessary. Some people like to write the number of the conditional first, and it's antecedent after. For instance, in proof 4 above I've justifed H on line 5 by 3,4 →out, but some people might prefer to write 4,3 →out. Either way is fine.

3. We're studying a formal system. One way to think of a *form*al system is that the only thing that's important is *form* or shape. In fact, all of our rules can be presented with p and q replaced by circles and squares. Sometimes students find it useful to put circles and squares within their proofs to help visualize the relations. Below is the →out rule presented with shapes, and a couple examples of how to think about the shapes within a proof.



1. (N)→[M]	A	
2. M→R	A	
3. (N)	A	
4. R→S	A	
5. [M]	1,3	→out
⋮		

1. (A→B)	A	
2. D→E	A	
3. C	A	
4. (A→B)→(C→D)	A	
5. [C→D]	1,4	→out
⋮		

Proofs • &out and &in

I. Construct proofs for the following arguments:

Easy

•1. B&R, D&S |– R&D
•2. C&M, (T&U)&V |– M&(U&C)
•3. (F&R)&K |– F&(R&K)
*•4. E&F, E→G, F→H |– G&H
•5. (R→S)&R, T&(T→W) |– W&S
•6. (W→T)&(S→N), S&W |– N&T
•7. (D&E)→H, D, E |– H
•8. R&N, S, (N&S)→I |– I&R
•9. (T&W)→Z, W&T |– Z

Medium

*•10. A&B, (A→D)&(B→E), (D&E)→F |– F
•11. J→(K&Z), (Z&K)→(O→S), J |– J&(O→S)
•12. T→(D&E), (E→(M&J))&T |– D&J
13. (M&N)→R, (N&M)→S, N, M |– R&(S&R)
14. S, S→G, C→H, ((S→G)&(C→H))→B |– G&B
•15. D, M&N, (D&M)→S, (D&N)→Z |– S&Z
•16. E, M&V, E→((V&M)→H), H→I |– I

Most Entertaining

•17. (A→J)&H, A&G, G→(H→M) |– (J&M)&(A→J)
•18. H&L, C, ((H&C)&L)→M |– M
•19. S, S→T, (T&S)→R, ((T&S)&R)→H |– H&(T&R)
•20. (F&M)→L, E&F, E→M, (E&(L&M))→R |– R
•21. [(A&B)&((D→E)&F)]→R, (B&A)&F, A→(D→E) |– R&(A→(D→E))
•22. (H&P)→M, B&(B→(P→D)), ((P→D)&B)→(P&H) |– M&(B&P)
*23. ((R→S)&T)→W, Z&(R→S), (S&T)→M, T&S |– T&(W&M)
•24. (P&Z)&H, (H&Z)→(F&M) |– [((H&Z)→(F&M))&F]&(P&M)
*25. (I&R)→N, A&(A→(R→E)), ((R→E)&A)→(R&I) |– N&(A&R)

Proof Rules for the First Test

automatic		creative
→out	**&**out	**&**in
p→q p —— q	p&q —— p q	p q —— p&q

Notes

1. For all of these rules, if you have what's above the "therefore bar," then you may write what's below it.
2. The automatic rules should be applied automatically–if you see the opportunity to use them, just do it. Creative rules should only be used with good reason.
3. Both of the ampersand rules are easy to use. When you have a formula with an & as main connective you use &out to break it up into two separate lines. When you want to build an & formula you combine two lines with &in.
4. Strictly speaking, &out is a "choice" rule; if you have p&q on a line in a proof, then you can write either p or q, whichever you want. In practice it usually makes sense to write them both, which is why I illustrate the rule this way.

• Solution next page.
* Video Online

Answers for page 1.11

1. C→(W→T)
or W→(C→T)
or (C&W)→T
or (W&C)→T
2. Same as 1
3. (D→H)→J
4. Same as 3
5. J→(B→I)
or B→(J→I)
or (J&B)→I
or (B&J)→I
6. (J→B)→I
7. D→(K→R)
or K→(D→R)
or (D&K)→R
or (K&D)→R
8. (T→E)→L
9. H→(R→S)
or R→(H→S)
or (H&R)→S
or (R&H)→S
10. (S→A)→P
11. G→(V→B)
or V→(G→B)
or (G&V)→B
or (V&G)→B
12. F→(R→A)
or R→(F→A)
or (F&R)→A
or (R&F)→A
13. (C→H)→A
14. (M→H)→W

Note: In many of the remaining answers I include optional parentheses in order to clarify structure.

15. (S&P)→((T&E)→(A&Z))
or (T&E)→((S&P)→(A&Z))
or ((T&E)&(S&P))→(A&Z)
or ((S&P)&(T&E))→(A&Z)
16. L→(N→(R&P))
or N→(L→(R&P))
or (L&N)→(R&P)
or (N&L)→(R&P)
17. ((M&L)→(J&H))→A
18. L→(W→(A&D&F))
or W→(L→(A&D&F))
or (L&W)→(A&D&F)
or (W&L)→(A&D&F)
19. (M&G)→((B&S)→C)
or (B&S)→((M&G)→C)
or ((M&G)&(B&S))→C
or ((B&S)&(M&G))→C
20. (S&Q&B)→((T&M)→P)
or (T&M)→((S&Q&B)→P)
or ((S&Q&B)&(T&M))→P
or ((T&M)&(S&Q&B))→P
21. (H→N)→(C&A)

Note: 22 and 23 include the second pattern as part of a longer sentence.

22. (B→F)&[C→(L→(R&S))]
or (B→F)&[L→(C→(R&S))]
or (B→F)&[(C&L)→(R&S)]
or (B→F)&[(L&C)→(R&S)]
23. (Y→((T&J)→E))&(C→A)
or ((T&J)→(Y→E))&(C→A)
or ((Y&(T&J))→E)&(C→A)
or [((T&J)&Y)→E]&(C→A)

Note: 24 and 25 are extensions of the second pattern with 4 parts.

24. M→(P→(O→B))
or (M&P&O)→B and others
25. S→(N→(G→B))
or (S&N&G)→B and others

1.

1. B&R	A
2. D&S	A
3. B	1 &out
4. R	" "
5. D	2 &out
6. S	" "
7. R&D	4,5 &in

2.

1. C&M	A
2. (T&U)&V	A
3. C	1 &out
4. M	" "
5. T&U	2 &out
6. V	" "
7. T	5 &out
8. U	" "
9. U&C	3,8 &in
10. M&(U&C)	4,9 &in

3.

1. (F&R)&K	A
2. F&R	1 &out
3. K	" "
4. F	2 &out
5. R	" "
6. R&K	3,5 &in
7. F&(R&K)	4,6 &in

4.

1. E&F	A
2. E→G	A
3. F→H	A
4. E	1 &out
5. F	" "
6. G	2,4 →out
7. H	3,5 →out
8. G&H	6,7 &in

5.

1. (R→S)&R	A
2. T&(T→W)	A
3. R→S	1 &out
4. R	" "
5. T	2 &out
6. T→W	" "
7. S	3,4 →out
8. W	5,6 →out
9. W&S	7,8 &in

6.

1. (W→T)&(S→N)	A
2. S&W	A
3. W→T	1 &out
4. S→N	" "
5. S	2 &out
6. W	" "
7. T	3,6 →out
8. N	4,5 →out
9. N&T	7,8 &in

7.

1. (D&E)→H	A
2. D	A
3. E	A
4. D&E	2,3 &in
5. H	1,4 →out

8.

1. R&N	A
2. S	A
3. (N&S)→I	A
4. R	1 &out
5. N	" "
6. N&S	2,5 &in
7. I	3,6 →out
8. I&R	4,7 &in

9.

1. (T&W)→Z	A
2. W&T	A
3. W	2 &out
4. T	" "
5. T&W	3,4 &in
6. Z	1,5 →out

10.

1. A&B	A
2. (A→D)&(B→E)	A
3. (D&E)→F	A
4. A	1 &out
5. B	" "
6. A→D	2 &out
7. B→E	" "
8. D	4,6 →out
9. E	5,7 →out
10. D&E	8,9 &in
11. F	3,10 →out

11.

1. J→(K&Z)	A
2. (Z&K)→(O→S)	A
3. J	A
4. K&Z	1,3 →out
5. K	4 &out
6. Z	" "
7. Z&K	5,6 &in
8. O→S	2,7 →out
9. J&(O→S)	3,8 &in

12.

1. T→(D&E)	A
2. (E→(M&J))&T	A
3. E→(M&J)	2 &out
4. T	" "
5. D&E	1,4 →out
6. D	5 &out
7. E	" "
8. M&J	3,7 →out
9. M	8 &out
10. J	" "
11. D&J	6,10 &in

15.

1. D	A
2. M&N	A
3. (D&M)→S	A
4. (D&N)→Z	A
5. M	2 &out
6. N	" "
7. D&M	1,5 &in
8. S	3,7 →out
9. D&N	1,6 &in
10. Z	4,9 →out
11. S&Z	8,10 &in

16.

1. E	A
2. M&V	A
3. E→((V&M))→H)	A
4. H→I	A
5. M	2 &out
6. V	" "
7. (V&M)→H	1,3 →out
8. V&M	5,6 &in
9. H	7,8 →out
10. I	4,9 →out

17.

1. (A→J)&H	A
2. A&G	A
3. G→(H→M)	A
4. A→J	1 &out
5. H	" "
6. A	2 &out
7. G	" "
8. H→M	3,7 →out
9. J	4,6 →out
10. M	5,8 →out
11. J&M	9,10 &in
12.(J&M)&(A→J)	4,11 &in

18.

1. H&L	A
2. C	A
3. ((H&C)&L)→M	A
4. H	1 &out
5. L	" "
6. H&C	2,4 &in
7. (H&C)&L	5,6 &in
8. M	3,7 →out

19.

1. S	A
2. S→T	A
3. (T&S)→R	A
4. ((T&S)&R)→H	A
5. T	1,2 →out
6. T&S	1,5 &in
7. R	3,6 →out
8. (T&S)&R	6,7 &in
9. H	4,8 →out
10. T&R	5,7 &in
11. H&(T&R)	9,10 &in

20.

1. (F&M)→L	A
2. E&F	A
3. E→M	A
4. (E&(L&M))→R	A
5. E	2 &out
6. F	" "
7. M	3,5 →out
8. F&M	6,7 &in
9. L	1,8 →out
10. L&M	7,9 &in
11. E&(L&M)	5,10 &in
12. R	4,11 →out

21.

1. [(A&B)&((D→E)&F)]→R	A
2. (B&A)&F	A
3. A→(D→E)	A
4. B&A	2 &out
5. F	" "
6. B	4 &out
7. A	" "
8. D→E	3,7 →out
9. A&B	6,7 &in
10. (D→E)&F	5,8 &in
11. (A&B)&((D→E)&F)	9,10 &in
12. R	1,11 →out
13. R&(A→(D→E))	3,12 &in

22.

1. (H&P)→M	A
2. B&(B→(P→D))	A
3. ((P→D)&B)→(P&H)	A
4. B	2 &out
5. B→(P→D)	" "
6. P→D	4,5 →out
7. (P→D)&B	4,6 &in
8. P&H	3,7 →out
9. P	8 &out
10. H	" "
11. H&P	9,10 &in
12. M	1,11 →out
13. B&P	4,9 &in
14. M&(B&P)	12,13 &in

24.

1. (P&Z)&H	A
2. (H&Z)→(F&M)	A
3. P&Z	1 &out
4. H	" "
5. P	3 &out
6. Z	" "
7. H&Z	4,6 &in
8. F&M	2,7 →out
9. F	8 &out
10. M	" "
11. ((H&Z)→(F&M))&F	2,9 &in
12. P&M	5,10 &in
13. [((H&Z)→(F&M))&F]&(P&M)	11,12 &in

Symbolizing Complex Conditionals • (p→q)→r vs. p→(q→r)

Symbolizing conditionals with multiple arrows is one of the trickier tasks in propositional logic. Success begins with understanding the difference between these two patterns:

(p→q)→r

Literally: If (if p then q) then r.
But best way to understand is to read as:

If (p *causes* q) then r.

p→(q→r) { equivalent to: (p&q)→r }

Literally: If p then (if q then r).
Best way to understand is to translate into & equivalent and read as:

If (both p and q are true) then r.

Recommended Strategy: When you come across complex conditionals try the following:

1. Identify the "r" part of the sentence, the final consequent.
2. Symbolize using both patterns, then read them following the suggestions above, and use intuitions to choose the correct pattern.

Examples

1. If, when you SING you're HAPPY, you should join a CHOIR.

"You should join a CHOIR" is the "r" part of the sentence. Thus we try these two symbolizations:

(S→H)→C ✔

Read it as:
If SINGING causes you to be HAPPY, then you should join a CHOIR.

Correct, because the original sentence is saying that the reason you should join a choir, is that there's a causal relation between singing and happiness.

S→(H→C)

Translate into (S&H)→C, and read it as:
If it's true that you're SINGING and you're HAPPY, then you should join a CHOIR.

Incorrect, because this symbolization says nothing about a relation between singing and happiness, but the original sentence does.

2. If you're HUNGRY, you should make a SANDWICH, supposing there's BREAD in the cupboard.

"You should make a SANDWICH" is the "r" part of the sentence. Thus we try these symbolizations:

(H→B)→S

Read it as:
If being HUNGRY causes there to be BREAD in the cupboard, then you should make a SANDWICH.

Incorrect, because the original sentence says nothing about hunger causing bread to exist in the cupboard; that would be silly.

H→(B→S) ✔

Translate into (H&B)→S, and read it as:
If it's true that you're HUNGRY and that there's BREAD in the cupboard, then you should make a SANDWICH.

Correct, because the original sentence is saying that the the reason to make a sandwich is that you're hungry AND that there's bread.

Notes:

1. Both of these patterns express different routes of "getting to" r. Thus the point of the recommended strategy is to identify r, and then use your intuitions to understand how you get there. Does r come about because p causes q? Or because p and q are both true?
2. Unfortunately, r does not always come at the end of the sentence. If you can't decide what counts as r try some symbolizations, and read them following the suggestions above to see if they make sense.
3. The logical synonyms "leads to," "results in," and "brings about" often appear in sentences of the first pattern.
4. Important: The arrow doesn't actually represent causation. As we'll see later in the semester, the correct interpretation of the arrow is not causal. Nonetheless, reading it as causes can be useful for some purposes.
5. The key to understanding the 2nd pattern is to recognize the equivalence between p→(q→r) and (p&q)→r, but since the & is commutative, it should be no surprise that another equivalence is (q&p)→r, and yet another is q→(p→r). In short, every time the 2nd pattern is correct, there are at least 4 different correct symbolizations.

Symbolizing Complex Conditionals • Exercises

$(p \rightarrow q) \rightarrow r$

If (p *causes* q) then r.

$p \rightarrow (q \rightarrow r)$ $\left\{ \begin{array}{c} \text{equivalent to:} \\ (p \& q) \rightarrow r \end{array} \right\}$

If (both p and q are true) then r.

I. Symbolize. (Answers in class or online.)

1. If you PLANT the seeds, they'll GROW, if you WATER them.
2. Provided that when students STUDY they do WELL, the class is FAIR.
3. If Socrates brings TABOULI and HUMMUS, then the philosophers will have a swell PARTY, provided that there is plenty of WINE and BAKLAVA.
4. You should read DESCARTES, KANT and NIETZSCHE, supposing that studying PHILOSOPHY leads to WISDOM and HAPPINESS.

II. Symbolize. The first 14 are straight forward examples of the two patterns above. (Answers on page 1.8.)

1. If she COMPETES, then if she WINS, then she will receive a TROPHY.
2. Assuming that she COMPETES, she'll receive a TROPHY if she WINS.
3. If, when you eat purple DESSERTS you experience HAPPINESS, then you ought to make grape JELLO.
4. You ought to make grape JELLO, if eating purple DESSERTS leads to HAPPINESS.
5. If JANET cleans the house, then if BRENDA buys beer, IRENE will throw a party.
6. If JANET'S cleaning the house results in BRENDA'S buying beer, then IRENE will throw a party.
7. If the DRESS is too small it may be RETURNED, as long as you KEEP the receipt.
8. Given that if you THINK too hard your brain will EXPLODE, you should do LOGIC homework outside.
9. Provided her voice is HEALTHY, she'll SING the song, assuming she REMEMBERS the words.
10. PLATO will be very surprised if SOCRATES' anger brings about ARISTOTLE'S apology.
11. You may join the BAND when you learn to play the GUITAR, assuming that we can use your VAN.
12. If there is FUR on the jello, then you should AVOID it, provided that you want to REMAIN standing.
13. If she HANGS up whenever you CALL, then you know she's ANGRY.
14. Given that he HOWLS when the MOON is full, he's probably a WEREWOLF.

The remaining sentences all involve additional complexity. Start by looking for obvious groupings.

15. If SOCRATES and PLATO play chess, then if the game is TENSE and EXCITING, ARISTOTLE and ZENO will watch intently.
16. You will be RICH and PROSPEROUS if you play the LOTTERY, so long as you choose the right NUMBERS.
17. If studying MATH and LOGIC results in JOY and HAPPINESS, then you should consider a job in ACADEMIA.
18. If Janet LOSES, she will be ANGRY, DISGUSTED and FLABBERGASTED if Brenda WINS.
19. In the event that the oceans turn into MILK and the deserts turn into GRANOLA, then if BOWLS and SPOONS fall from the sky, the whole world will eat CEREAL.
20. Supposing that I am in the mood for SNAILS, QUICHE and BURGUNDY, then I will fly to PARIS, provided that I have the TIME and the MONEY.
21. Audiences CHEER and APPLAUD, when HEROIC deeds result in NOBLE achievements.
22. A BUSINESS degree results in FINANCIAL success, nevertheless, given a CHOICE I would RISK poverty and STUDY philosophy, provided that I can pass LOGIC.
23. If it's YELLOW, then if it's TRANSLUCENT and JIGGLY, it's EDIBLE, but if it's CRUSTY it should be AVOIDED.

◊24. When playing MONOPOLY, if you land on a PROPERTY, you may BUY it, if it's OWNED by the bank.

◊25. Supposing it's SUNDAY, then if it's NOON, I'll be at the BEACH, provided that the weather is GOOD.

I. Say whether the following arguments are **sound**, **valid but not sound**, or **invalid**. (Answers next page)

1. If Opus is a penguin, then Opus is a bird. (T)
   Opus is a penguin. (T)
   Opus is a bird.

2. If Opus is a penguin, then Opus is a bird. (T)
   Opus is a bird. (T)
   Opus is a penguin.

3. If Opus is a bird, then Opus is a penguin. (F)
   Opus is a bird. (T)
   Opus is a penguin.

4. If Opus is a bird, then Opus is a penguin. (F)
   Opus is a penguin. (T)
   Opus is a bird.

5. If Opus is a penguin, then Opus can fly. (F)
   Opus is a penguin. (T)
   Opus can fly.

6. If Opus is a penguin, then Opus can swim. (T)
   Opus is a penguin. (T)
   Opus can swim.

7. If Opus is a human, then Opus enjoys opera. (F)
   Opus is a human. (F)
   Opus enjoys opera.

8. If Opus is a human, then Opus enjoys opera. (F)
   Opus enjoys opera. (F)
   Opus is a human.

II. Say whether the following are true or false. (Answers next page)

1. ___ Some sound arguments are invalid.
2. ___ Some invalid arguments are sound.
3. ___ Some valid arguments have false premises.
4. ___ All sound arguments have true premises.
5. ___ All sound arguments are valid.
6. ___ All valid arguments are sound.
7. ___ All arguments with false premises are invalid.
8. ___ Some arguments are true.

III. Symbolize the following sentences. (Answers next page.)

1. If the PHILOSOPHERS throw a party, the MUSIC will be lousy, but the JELLO will be delicious.
2. Although René Descartes was a PHILOSOPHER, MATHEMATICIAN, and SCIENTIST, he is best known as a DREAMER and the potential VICTIM of a demonic genius.
3. JANET, BRENDA and IRENE will drink malt liquor, provided that they've finished the jello SHOTS.
4. Should the ASTRONOMERS solve the problem then the PHYSICISTS will be jealous, however, the MATHEMATICIANS will be humiliated if it is solved by the LOGICIANS.
5. Though Brenda spent the afternoon READING philosophy and CLEANING the house, Janet was LOUNGING on the sofa and NIBBLING cheetos while ENJOYING the soothing sounds of Lawrence Welk.
6. Given that SOCRATES was a philosopher, he was likely to be THOUGHTFUL and INTROSPECTIVE, but also socially AWKWARD and prone to DRINKING large quantities of cheap beer.
7. If it's the case that when the economy IMPROVES the stock market RISES, then if the economy improves you ought to BUY stocks. (Reminder: Repeated components are only capitalized in first appearance.)
8. Janet is making JELLO and PUDDING, and she will GIVE you some if you ASK her nicely.
9. Even though PLATO and ARISTOTLE thought the tabouli was tasty, if they'd had a CHOICE they would have preferred pita BREAD and a bucket of HUMMUS.
10. While it's true that if you're HAPPY and you KNOW it you should CLAP your hands, you should STOMP your feet if you're ANGRY and BEWILDERED.

The remaining sentences are complex conditionals like those on pages 1.10 and 1.11

11. Supposing that the WORLD will end tomorrow, then we should do our LOGIC homework tonight if we want to get it FINISHED.
12. I'll have to WALK and ride a BICYCLE, as long as STEALING cars results in going to JAIL.
13. If the SALSA is hot and the CERVEZA is cold, then we will have a swell FIESTA, provided that the MARIACHIS are good and the PIÑATA is large.
14. Supposing that if you eat too much ICE cream and CAKE you'll get SICK and DIE, then you ought to be careful at BIRTHDAY parties.
15. If the FOOD is good she will say YES, provided that the TIMING is right and the DIAMOND is big enough.
16. Assuming that MARZIPAN grows on trees and FIREFLIES live underwater, then if SALAMANDERS waltz in the moonlight I will find a BILLION dollars in the cushions of my sofa and there will be world PEACE.

IV. Construct proofs for the following arguments. (Answers next page)

1. $(A\&B)\rightarrow E, (F\rightarrow A)\&(H\rightarrow B), F\&H \vdash E$
2. $((R\&S)\&(T\&Z))\rightarrow W, (T\&R)\rightarrow(S\&Z), R\&T \vdash W$
3. $S\&P, Q\rightarrow R, P\rightarrow(S\rightarrow T), ((Q\rightarrow R)\&T)\rightarrow M \vdash M\&(T\&(Q\rightarrow R))$
4. $(S\rightarrow R)\&(Z\rightarrow(W\rightarrow T)), (E\&G)\rightarrow(Z\&W), (Z\rightarrow(W\rightarrow T))\rightarrow(G\&E) \vdash T\&(S\rightarrow R)$
5. $(A\rightarrow B)\rightarrow C, A, (A\rightarrow B)\&(D\rightarrow E), ((A\rightarrow B)\&(D\rightarrow E))\rightarrow F, ((A\rightarrow B)\rightarrow C)\rightarrow D \vdash ((A\&B)\&(C\&D))\&(E\&F)$
6. $(H\rightarrow S)\rightarrow(T\rightarrow(D\&H)), T\&(H\rightarrow S), (T\&(H\&D))\rightarrow J \vdash J\&S$
7. $(R\&((A\rightarrow B)\rightarrow D))\rightarrow M, A\rightarrow(B\rightarrow D), (A\rightarrow B)\rightarrow D, (A\&R)\&B, ((M\&(B\rightarrow D))\&D)\rightarrow S \vdash S$

Section I.
1. Sound
2. Invalid
3. Valid but not sound.
4. Invalid.
5. Valid but not sound.
6. Sound
7. Valid but not sound.
8. Invalid.

Section II.
1. False
2. False
3. True
4. True
5. True
6. False
7. False
8. False

Section III. Symbolization
1. P→(M&J)
2. (P&M&S)&(D&V)
or P&M&S&D&V
3. S→(J&B&I)
4. (A→P)&(L→M)
5. (R&C)&(L&N&E)
or R&C&L&N&E
6. S→((T&I)&(A&D))
or S→(T&I&A&D)
7. (I→R)→(I→B)
8. (J&P)&(A→G)
or J&P&(A→G)
9. (P&A)&(C→(B&H))
or P&A&(C→(B&H))
10. ((H&K)→C)&((A&B)→S)
11. W→(F→L)
or F→(W→L)
or (W&F)→L
or (F&W)→L
12. (S→J)→(W&B)
13. (S&C)→((M&P)→F)
or (M&P)→((S&C)→F)
or ((S&C)&(M&P))→F
or (S&C&M&P)→F
14. ((I&C)→(S&D))→B
15. F→((T&D)→Y)
or (T&D)→(F→Y)
or (F&(T&D))→Y
or ((T&D)&F)→Y
16. (M&F)→(S→(B&P))
or S→((M&F)→(B&P))
or ((M&F)&S)→(B&P)
or (S&(M&F))→(B&P)

Section IV. Proofs

1.

1. (A&B)→E	A
2. (F→A)&(H→B)	A
3. F&H	A
4. F→A	2 &out
5. H→B	" "
6. F	3 &out
7. H	" "
8. A	4,6 →out
9. B	5,7 →out
10. A&B	8,9 &in
11. E	1,10 →out

2.

1. ((R&S)&(T&Z))→W	A
2. (T&R)→(S&Z)	A
3. R&T	A
4. R	3 &out
5. T	" "
6. T&R	4,5 &in
7. S&Z	2,6 →out
8. S	7 &out
9. Z	" "
10. R&S	4,8 &in
11. T&Z	5,9 &in
12. (R&S)&(T&Z)	10,11 &in
13. W	1,12 →out

3.

1. S&P	A
2. Q→R	A
3. P→(S→T)	A
4. ((Q→R)&T)→M	A
5. S	1 &out
6. P	" "
7. S→T	3,6 →out
8. T	5,7 →out
9. (Q→R)&T	2,8 &in
10. M	4,9 →out
11. T&(Q→R)	2,8 &in
12. M&(T&(Q→R))	10,11 &in

4.

1. (S→R)&(Z→(W→T))	A
2. (E&G)→(Z&W)	A
3. (Z→(W→T))→(G&E)	A
4. S→R	1 &out
5. Z→(W→T)	" "
6. G&E	3,5 →out
7. G	6 &out
8. E	" "
9. E&G	7,8 &in
10. Z&W	2,9 →out
11. Z	10 &out
12. W	" "
13. W→T	5,11 →out
14. T	12,13 →out
15. T&(S→R)	4,14 &in

5.

1. (A→B)→C	A
2. A	A
3. (A→B)&(D→E)	A
4. ((A→B)&(D→E))→F	A
5. ((A→B)→C)→D	A
6. A→B	3 &out
7. D→E	" "
8. B	2,6 →out
9. C	1,6 →out
10. D	1,5 →out
11. F	3,4 →out
12. E	7,10 →out
13. A&B	2,8 &in
14. C&D	9,10 &in
15. E&F	11,12 &in
16. (A&B)&(C&D)	13,14 &in
17. ((A&B)&(C&D))&(E&F)	15,16 &in

6.

1. (H→S)→(T→(D&H))	A
2. T&(H→S)	A
3. (T&(H&D))→J	A
4. T	2 &out
5. H→S	" "
6. T→(D&H)	1,5 →out
7. D&H	4,6 →out
8. D	7 &out
9. H	" "
10. S	5,9 →out
11. H&D	8,9 &in
12. T&(H&D)	4,11 &in
13. J	3,12 →out
14. J&S	10,13 &in

7.

1. (R&((A→B)→D))→M	A
2. A→(B→D)	A
3. (A→B)→D	A
4. (A&R)&B	A
5. ((M&(B→D))&D)→S	A
6. A&R	4 &out
7. B	" "
8. A	6 &out
9. R	" "
10. B→D	2,8 →out
11. D	7,10 →out
12. R&((A→B)→D)	3,9 &in
13. M	1,12 →out
14. M&(B→D)	10,13 &in
15. (M&(B→D))&D	11,14 &in
16. S	5,15 →out